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The construction is effected as follows: let $V_1V_2V_3$ be the given triangle. Let X be so located that

$$\measuredangle C_2XC_3 = \measuredangle C_2MC_3 + \measuredangle V_3V_1V_2$$

with similar expressions for $\measuredangle C_3XC_1$ and $\measuredangle C_1XC_2$. There is just one such point X. Now regard circle C_2XC_3 as a part of Fig. 1, and find the homologous circle of II, then P_2, one of the required triad of points, lies on this circle. But P_2 lies also on circle C_3XC_1, and therefore it is uniquely determined.

THEOREM 17. *Similarly, there is a single triad of collinear homologous points, whose distances are in the same ratios as the distances among three given collinear points.*[1]

It is well known that in the special case when three similar figures are constructed on the sides of a triangle $A_1A_2A_3$, in such a way that A_2A_3, A_3A_1, A_1A_2 are homologous segments, then

(*a*) the triangle of similitude is the second Brocard triangle,
(*b*) the invariable triangle is the first Brocard triangle,
(*c*) the circle of similitude is therefore the Brocard circle,
(*d*) the point M is the median point,
(*e*) the circles MC_2C_3, etc., are the Neuberg circles.

NOTE ON A CERTAIN CLASS OF DETERMINANTS.

By W. H. METZLER, Syracuse University.

In this journal for May, 1915, there appeared the translation of a paper by Professor Pascal which considered a class of determinants which have the property of being expressible as the sum of two squares, and it is the main object of this note to point out another proof of this fact.

The determinants in question are of the form:

$$D \equiv \begin{vmatrix} a_{11} & \cdots & a_{1n} & -b_{11} & \cdots & -b_{1n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{n1} & \cdots & a_{nn} & -b_{n1} & \cdots & -b_{nn} \\ b_{11} & \cdots & b_{1n} & a_{11} & \cdots & a_{1n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ b_{n1} & \cdots & b_{nn} & a_{n1} & \cdots & a_{nn} \end{vmatrix},$$

which is the determinant of the set of linear homogeneous complex equations for the $2n$ quantities $p_1, \cdots, p_n, q_1, \cdots, q_n$:

$$\begin{array}{c} (a_{11} + b_{11}\cdot i)(p_1 + q_1\cdot i) + \cdots + (a_{1n} + b_{1n}\cdot i)(p_n + q_n\cdot i) = 0 \\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot\ \cdot \\ (a_{n1} + b_{n1}\cdot i)(p_1 + q_1\cdot i) + \cdots + (a_{nn} + b_{nn}\cdot i)(p_n + q_n\cdot i) = 0. \end{array} \qquad (1)$$

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[1] Theorems 16 and 17 are doubtless well known, but the author has not chanced to see them explicitly stated. But see McClelland.

The determinant D is obviously unaltered in value[1] by rearranging the last n rows and the last n columns so that they will be in the reverse order, and then multiplying the elements of the first n rows and the last n columns by i. In this form it is seen to be centro-symmetric with all the b's positive and all the a's multiplied by i. It follows, therefore, that it is the product of two factors D' and D'', say, and it has been shown[2] that one of these factors is the sum of two sets of terms and the other is the difference of the same two sets.

Thus

$$D' \equiv \begin{vmatrix} b_{1n} + a_{1n}\cdot i & \cdots & b_{11} + a_{11}\cdot i \\ \cdot\;\cdot\;\cdot\;\cdot & \cdot\;\cdot\;\cdot & \cdot\;\cdot\;\cdot\;\cdot\;\cdot \\ b_{nn} + a_{nn}\cdot i & \cdots & b_{n1} + a_{n1}\cdot i \end{vmatrix}, \qquad \text{or} \qquad | \, b_{rs} + a_{rs}\cdot i \, | \, ;$$

and

$$D'' \equiv \begin{vmatrix} b_{1n} - a_{1n}\cdot i & \cdots & b_{11} - a_{11}\cdot i \\ \cdot\;\cdot\;\cdot\;\cdot & \cdot\;\cdot\;\cdot & \cdot\;\cdot\;\cdot\;\cdot\;\cdot \\ b_{nn} - a_{nn}\cdot i & \cdots & b_{n1} - a_{n1}\cdot i \end{vmatrix}, \qquad \text{or} \qquad | \, b_{rs} - a_{rs}\cdot i \, | \, .$$

As D' and D'' are determinants with binomial elements, each may be expanded into a sum of determinants with monomial elements giving

$$\begin{aligned} D &= (X + i\cdot Y)(X - i\cdot Y) \\ &= X^2 + Y^2. \end{aligned}$$

Representing the matrix of D symbolically by

$$\left\| \begin{array}{cc} (A) & (B) \\ (-B) & (A) \end{array} \right\|,$$

Professor Pascal remarks that "it still remains to study determinants whose matrices are of the type

$$\left\| \begin{array}{cccc} (A) & (B) & (C) & \cdots \\ (-B) & (A) & (D) & \cdots \\ (-C) & (-D) & (A) & \cdots \\ \cdot\;\cdot & \cdot\;\cdot & \cdot\;\cdot & \cdot\;\cdot \end{array} \right\|."$$

If our set of equations were of the form:

$$\begin{aligned} (a_{11} + b_{11}\cdot i)(p_1 + q_1\cdot i) + \cdots + (a_{1n} + b_{1n}\cdot i)(p_n + q_n\cdot i) &= 0, \\ -(a_{12} + b_{12}\cdot i)(p_1 + q_1\cdot i) + \cdots + (a_{2n} + b_{2n}\cdot i)(p_n + q_n\cdot i) &= 0, \qquad (2) \\ \cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot\;\cdot & \\ -(a_{1n} + b_{1n}\cdot i)(p_1 + q_1\cdot i) - \cdots + (a_{11} + b_{11}\cdot i)(p_n + q_n\cdot i) &= 0, \end{aligned}$$

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[1] The sign will be changed if n is odd.

[2] Metzler, On Centro-Symmetric and Skew-Centro-Symmetric Determinants, *Messenger of Mathematics*, New Series, No. 515, March, 1914.

where the diagonal coefficients are all $a_{11} + b_{11}i$, the determinant D would be

$$D = \begin{vmatrix} a_{11} & -b_{11} & a_{12} & -b_{12} & \cdots & a_{1n} & -b_{1n} \\ b_{11} & a_{11} & b_{12} & a_{12} & \cdots & b_{1n} & a_{1n} \\ -a_{12} & b_{12} & a_{11} & -b_{11} & \cdots & a_{2n} & -b_{2n} \\ -b_{12} & -a_{12} & b_{11} & a_{11} & \cdots & b_{2n} & a_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ -a_{1n} & b_{1n} & -a_{2n} & b_{2n} & \cdots & a_{11} & -b_{11} \\ -b_{1n} & -a_{1n} & -b_{2n} & -a_{2n} & \cdots & b_{11} & a_{11} \end{vmatrix};$$

or expressed symbolically

$$D = \left\| \begin{array}{cccc} (A) & (B) & (C) & \cdots \\ (-B) & (A) & (D) & \cdots \\ (-C) & (-D) & (A) & \cdots \\ \cdot & \cdot & \cdot & \cdot \end{array} \right\|,$$

the form in question.

By a rearrangement of rows and columns it is readily seen that D may be represented symbolically by

$$\left\| \begin{array}{cc} (A') & (B') \\ (-B') & (A') \end{array} \right\|.$$

This is what should be expected since the D for equations (2) is a particular case of the D for equations (1).

If the A, B, C, $\cdots$ here are to be of higher order than the second it will be necessary to take equations from algebras of higher order, such as quaternions for the fourth order.

THE MARYLAND-VIRGINIA-DISTRICT OF COLUMBIA SECTION.

The Maryland-Virginia-District of Columbia Section of the Mathematical Association of America held its fall meeting at St. John's College, Annapolis, Md., Saturday, December 15, 1917. Among the thirty-two persons in attendance were the following members of the Association:

O. S. Adams, U. S. Coast and Geodetic Survey, Washington, D. C.
J. A. Bullard, U. S. Naval Academy, Annapolis, Md.
Paul Capron, U. S. Naval Academy, Annapolis, Md.
G. R. Clements, U. S. Naval Academy, Annapolis, Md.
A. B. Coble, Johns Hopkins University, Baltimore, Md.
A. Cohen, Johns Hopkins University, Baltimore, Md.
J. B. Eppes, U. S. Naval Academy, Annapolis, Md.
J. N. Galloway, U. S. Naval Academy, Annapolis, Md.
H. C. Gossard, U. S. Naval Academy, Annapolis, Md.
Angelo Hall, U. S. Naval Academy, Annapolis, Md.